



## Technical Note

# Emissive power of semitransparent spherical particle with nonuniform temperature

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**Abstract**

The ray tracing method and the zonal method is extended to study the emissive power of semitransparent spherical particle with nonuniform temperature. The particle emissive power is calculated by the radiative transfer coefficients. The effects of the related parameters on the particle emissive power and the errors resulted from omitting the non-uniformity of particle temperature are analyzed and discussed. The results show that omitting the nonuniformity of particle temperature will result in large errors of particle emissive power, and the errors increase with the nonuniformity of particle temperature. The particle emissive power based on the average temperature may deviate from its real value, and the size parameter and absorption index strongly affects the ratio of particle emissive power based on the average temperature and its real value.

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*Keywords:* Semitransparent medium; Spherical particle; Radiative energy; Ray tracing method

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**1. Introduction**

Semitransparent particles will be the main radiative participating media in many high-temperature systems. The temperature distribution within semitransparent particle is often nonuniform and strongly affected by physical and chemical processes inside or outside the particle. For many engineering practices, particle emissive power is often calculated by particle average temperature. However, due to the nonlinearity of Stefan–Boltzmann's law and Planck's law, the emissive power based on the particle average temperature may deviate significantly from its real value, and so, omitting the nonuniformity of particle temperature will result in errors for the solution of radiative heat transfer. Recently, a lot of work has been reported on the radiative heat transfer in par-

ticulate systems. Luan et al. [1] studied the radiative heat transfer in circulating fluidized beds by discrete ordinates method. Caldas and Semiao [2] analyzed the modeling of scattering and absorption coefficients for a polydispersion. Marakis et al. [3] made a parametric study on radiative heat transfer in pulverized coal furnaces. Wu and Chu [4] studied the combined conduction and radiation heat transfer in plane-parallel packed beds with variable porosity. All these work didn't consider the effects of nonuniformity of particle temperature on calculating particle emissive power in particulate systems.

We have studied the internal distribution of radiative absorption in one-dimensional slab, sphere and cylinder under the irradiation of surroundings by the ray tracing method [5–8], and found that the internal distribution of spectral radiative absorption in spherical and cylindrical particles differs from that in slab, the peak may locate at interior shell of the semitransparent sphere and cylinder. The dimensionless volumetric spectral radiative absorption is higher near the center for weakly absorbing

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Nomenclature			
$k$	absorption index	$[V_i S]_\lambda$	spectral radiative transfer coefficient of volume $i$ vs. surrounding
$m$	complex index of refraction, $m = n - ik$	$x$	size parameter, $x = 2\pi R/\lambda$
$n$	refractive index	$x_{\max}$	size parameter, $x_{\max} = 2\pi RT_{\text{av}}/2897.8$
$Q$	particle emissive power	$y$	dimensionless radial coordinate, $y = r/R$
$Q^*$	particle emissive power based on average temperature	$\beta$	ratio of particle emissive power, $\beta = Q/Q^*$
$R$	particle radius	$\theta$	incident angle
$r$	radial coordinate	$\phi$	refraction angle
$[SV_i]_\lambda$	spectral radiative transfer coefficient of surrounding vs. volume $i$	$\lambda$	wavelength in vacuum
$T$	particle temperature	$\rho_{\parallel}$	parallel polarized component of reflectivity
$T_{\text{av}}$	particle average temperature	$\rho_{\perp}$	perpendicular polarized component of reflectivity

or small spheres and cylinders, but higher near the surface for strongly absorbing or large spheres and cylinders [6]. Because emission is the inverse process of absorption, according to the principle of detailed equilibrium, the volumetric contribution of different local sites to emissive power is nonuniform, it is higher near the center for weakly absorbing or small particle, but higher near the surface for strongly absorbing or large particle. Therefore, the particle emissive power depends on the temperature distribution and its uniformity within particle.

The objective of the present work is to extend the ray tracing method and the zonal method to study the emissive power of semitransparent spherical particle with nonuniform temperature distribution. The radiative transfer coefficients are deduced by use of the ray tracing method in combination with the zonal method. The particle emissive power is calculated by the radiative transfer coefficients. The effects of the related parameters on the particle emissive power and the errors resulted from omitting the nonuniformity of particle temperature will be analyzed and discussed.

**2. Physical model and formulation**

A semitransparent spherical particle with radius  $R$  is considered. The following assumptions are made in the analysis: (1) the particle is composed of isotropic and homogeneous medium; (2) the medium emits and absorbs but does not scatter thermal radiation; (3) the particle surface is optically smooth; and (4) the complex index of refraction,  $m = n - ik$ , does not depend particle temperature  $T$  and wavelength  $\lambda$ .

We use the ray tracing method to deduce the formulae, for computing the emissive power of particle with nonuniform temperature distribution. As shown in Fig. 1, the semitransparent spherical particle is divided into  $M$  uniform control volume elements in radial di-

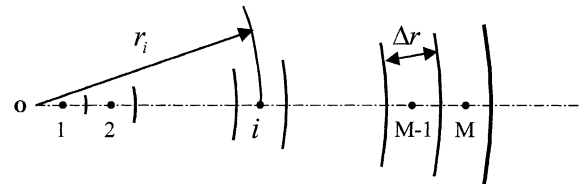


Fig. 1. Grid system.

rection. The spectral radiative transfer coefficient of surrounding vs. volume  $i$ ,  $[SV_i]_\lambda$ , can be expressed as [8]

$$[SV_i]_\lambda = G_\lambda(y + 0.5 \Delta y) - G_\lambda(y - 0.5 \Delta y) \tag{1}$$

where

$$G_\lambda(y) = 4\pi R^2 \int_0^{\theta_{\text{end}}} \exp \left[ -2xk \left( \cos \phi - \sqrt{y^2 - \sin^2 \phi} \right) \right] - \exp \left[ -2xk \left( \cos \phi + \sqrt{y^2 - \sin^2 \phi} \right) \right] \left\{ \frac{1 - \rho_{\perp}(\theta)}{1 - \rho_{\perp}(\theta) \exp(-4xk \cos \phi)} + \frac{1 - \rho_{\parallel}(\theta)}{1 - \rho_{\parallel}(\theta) \exp(-4xk \cos \phi)} \right\} \sin \theta \cos \theta d\theta \tag{2}$$

Here,  $y = r/R$  is the dimensionless radial coordinate,  $x = 2\pi R/\lambda$  is the size parameter,  $\theta$  is the incident angle,  $\phi$  is the refraction angle, and  $\theta_{\text{end}}$  is determined as

$$\theta_{\text{end}} = 0.5\pi, \quad \text{if } \frac{1}{n} < y \leq 1 \tag{3a}$$

$$\theta_{\text{end}} = \sin^{-1}(ny), \quad \text{if } 0 \leq y \leq \frac{1}{n} \tag{3b}$$

The detail deducing procedure of parameter  $G_\lambda(y)$  can be seen in Ref. [8], and will not repeated here. According to the relativities of the spectral radiative transfer coeffi-

cient, the transfer coefficient of volume  $i$  vs. surrounding is equal to that of surrounding vs. volume  $i$ , i.e.,

$$[SV_i]_\lambda = [V_iS]_\lambda \tag{4}$$

By using of the spectral radiative transfer coefficient, the emissive power emitting by a particle with nonuniform temperature can be written as

$$Q = \sum_{i=1}^M \int_0^\infty [V_iS]_\lambda \pi I_{b\lambda}(T_i) d\lambda \tag{5}$$

Similarly, if the nonuniformity of temperature within particle is omitted, the particle emissive power based on mass average temperature of particle,  $T_{av}$ , can be written as

$$Q^* = \sum_{i=1}^M \int_0^\infty [V_iS]_\lambda \pi I_{b\lambda}(T_{av}) d\lambda \tag{6}$$

For the sake of analysis and comparison, the ratio of emissive power,  $\beta$ , is introduced as

$$\beta = Q/Q^* \tag{7}$$

### 3. Results and discussion

A computer code of numerical integration was written based on the above formulation. Grid refinement studies were also performed for the numerical integration to ensure that the essential physics are independent of grid size. Because the geometrical optics is valid only if the size parameter of particle is larger than the wavelength of rays, we only consider here the emissive power of the particle with size parameter  $x_{max} \geq 30$ , i.e. the size parameter  $x_{max}$  is based on the wavelength corresponding to the maximum spectral emissive power

of blackbody at particle average temperature, and can be written as

$$x_{max} = \frac{2\pi RT_{av}}{2897.8} \tag{8}$$

Fig. 2 shows the effects of nonuniformity of particle temperature on the particle emissive power. The temperature distribution is assumed to be linear within particle and given by

$$T(y) = 1000 + a(0.5 - y), \text{ K} \tag{9}$$

where  $a$  is the parameter denoting nonuniformity of particle temperature. The nonuniformity of particle temperature increases with  $|a|$ . When  $a = 0$ , the temperature within particle is uniform. As shown in Fig. 2, omitting the nonuniformity of particle temperature results in large errors of particle emissive power, and the errors increase with the nonuniformity of particle temperature. With increases of  $a$ , the temperature decreases near the particle surface, and increases near the particle center. In the case of  $x_{max} = 300$  and  $k = 0.1$ , the particle

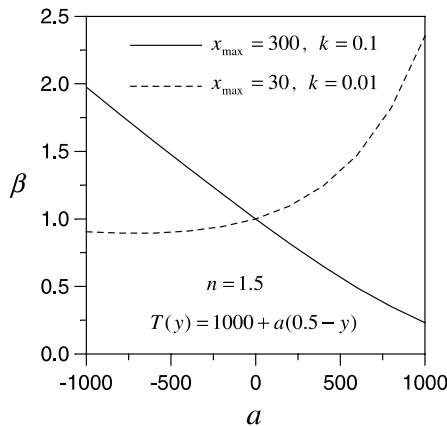


Fig. 2. Effects of nonuniformity of temperature on the particle emissive power.

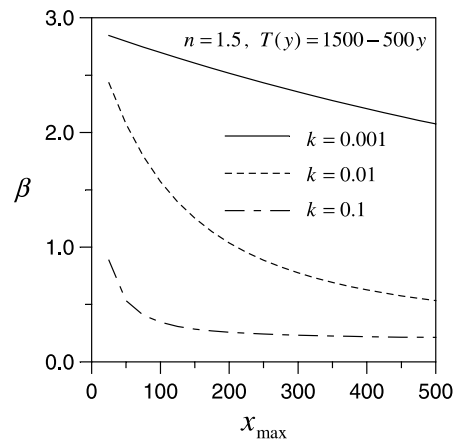
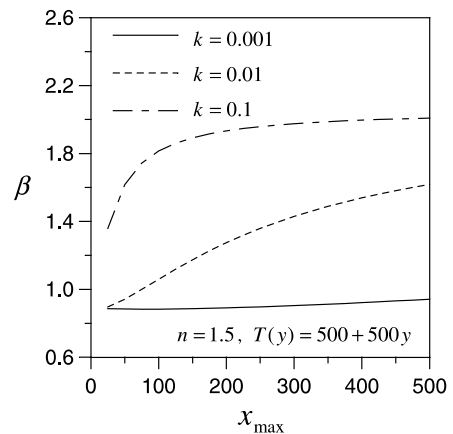


Fig. 3. Effects of size parameter and absorption index on the particle emissive power.

is strongly absorbing particle. The local volume contribution to emissive power is higher near the surface. Therefore, with increases of  $a$ , the ratio of emissive power decreases. In the case of  $x_{\max} = 30$  and  $k = 0.01$ , the particle is weakly absorbing. The local volume contribution to emissive power is higher near the particle center. Therefore, with increases of  $a$ , the ratio of emissive power increases.

Fig. 3 shows the effects of size parameter and absorption index on the particle emissive power, from which the size parameter and absorption index strongly affects the ratio of particle emissive power, and the particle emissive power based on the average temperature can be larger or less than its real value. When the temperature near particle surface is higher than that near particle center (see Fig. 3(a)), the ratio of particle emissive power,  $\beta$ , increases with the size parameter and the absorption index. But when the temperature near particle surface is lower than that near particle center (see Fig. 3(b)), the ratio of particle emissive power decreases with increases of the size parameter and the absorption index.

#### 4. Conclusions

The ray tracing method and the zonal method are used to study the emissive power of semitransparent spherical particle with nonuniform temperature. The effects of the related parameters on the particle emissive power and the errors resulted from omitting the nonuniformity of particle temperature are analyzed and discussed. The main conclusions can be summarized as follows:

1. Omitting the nonuniformity of particle temperature will result in large errors of particle emissive power, and the errors increase with the nonuniformity of particle temperature.
2. The size parameter and the absorption index strongly affect the ratio of particle emissive power, and the particle emissive power based on the average temperature can be larger or less than its real value.
3. When the temperature near particle surface is higher than that near particle center, the ratio of particle

emissive power increases with the size parameter and the absorption index. But when the temperature near particle surface is lower than that near particle center, the ratio of particle emissive power decreases with increases of the size parameter and the absorption index.

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